ECON 381 Midterm #1

Prof. Krauth, Spring 2013

Please answer the question asked. You will be graded exclusively on your answer – you will receive no credit for extraneous information or almost getting it right and you will receive no penalty for failing to "show your work." Answers written in pencil will not be eligible for regrade.

I will use the phrase "standard preferences" to mean that:

- All goods are normal.
- No two goods are perfect substitutes for each other.
- No two goods are perfect complements for each other.

I will use the phrase "standard production function" to mean that:

- The firm's output *Q* is determined by the continuous production function *Q=F(N,K)*, where *N* is labour and *K* is capital.
- The production function exhibits positive but declining marginal product in both capital and labour.

Unless I say otherwise, use *C* for consumption, *L* for leisure, *N* for labour, *w* for the wage, *p* for the output price, and *Q* for the output quantity.

Here is a convenient result for calculating elasticities:

$$\frac{d \ln y}{d \ln x} = \frac{dy}{dx} * \frac{x}{y}$$

as long as all of those derivatives exist.

True, false or uncertain (5 points each)

For each of these statements please indicate whether the statement is true, false, or uncertain, and briefly (1-3 sentences should do) explain why. Your grade will depend primarily on the quality of your explanation.

1. For a worker that chooses to work, the wage is equal to the marginal rate of substitution between income and leisure.
2. A fixed annual payment of $1,000 to each Canadian would have no impact on labour supply.
3. We can measure the effect of social assistance on labour supply by comparing employment rates in states or provinces with generous social assistance programs to employment rates in states or provinces with less generous programs.
4. The EI system, which is characterized by payments to unemployed workers who have worked in at least 20 out of the past 52 weeks, can actually lead to increased labour supply by some workers.

5. ICBC is the exclusive supplier of liability insurance to drivers in British Columbia. As a result, we would expect ICBC's labour demand to be more elastic than that of most other firms.
6. The labour supply of men is quite elastic.
7. An increase in the cost of agricultural land or equipment will reduce the wages of farm workers.
8. Nonwage benefits have grown over the past 50 years in absolute terms, but have shrunk as a proportion of total compensation.
9. Recurring quasi-fixed costs such as payroll taxes for CPP/QPP, EI and workers' compensation premiums lead to labour hoarding.
10. Layoff rates are higher among low-skilled workers because their hiring and training costs are lower.

Problems

1. **(26 points)** Suppose that the firm has *K* units of capital and hires *N* units of labour at wage *w* to produce *Q* units of its product according to the Cobb-Douglas production function:

$$Q = zN^{a}K^{1-a}$$

It then sells its good at price *p*. The firm takes wages and output prices as given, and takes *K* as fixed.

   a. Find the elasticity of the firm's output with respect to labour, i.e.

$$\eta_Q = \frac{dQ}{dN} * \frac{N}{Q}$$

   as a function of[1] (*a,z,N,K*).
   b. Taking capital and prices as given, find the firm's marginal revenue product of labour (*MRPL*) as a function of (*p,a,z,N,K*)
   c. Suppose that *p* is high enough relative to *w* that the firm has nonzero labour demand. Find the firm's labour demand curve N^D as a function of (*p,a,z,w,K*).
   d. Find the elasticity of labour demand with respect to wages:

$$\eta_D = \frac{dN^D}{dw} * \frac{w}{N^D}$$

   as a function of (*p,a,z,K*).
   e. Find the elasticity of labour demand with respect to the output price:

[1] When I say "as a function of", I am giving you a list of variables you are allowed to include in your answer. Not all of those variables are necessarily in your answer.

$$\eta_p = \frac{dN^D}{dp} * \frac{p}{N^D}$$

as a function of (*p,a,z,K*).

f. Find the elasticity of labour demand with respect to capital:

$$\eta_K = \frac{dN^D}{dK} * \frac{K}{N^D}$$

as a function of (*p,a,z,K*).

g. Find the marginal rate of technical substitution (MRTS), i.e.:

$$MRTS = \frac{{}^{dQ}/_{dN}}{{}^{dQ}/_{dK}}$$

as a function of (*a,z,N,K*).

h. Now suppose we are in the long run. K is now variable, and can be rented from the market at rental rate r. Suppose that the firm has chosen to produce quantity Q_1. The cost-minimizing mix of capital and labour will equalize the MRTS and the relative prices, i.e.:

$$\frac{w}{r} = MRTS$$

Find the firm's desired capital per worker (K/N) as a function of (*a,z,w,r*).

2. **(24 points)** We have talked about the influence of quasi-fixed costs (in the form of nonwage benefits) on labour demand. This question considers their impact on labour supply.

Consider the weekly labour supply decision of a single father who receives Y_N per week in nonlabour income and has up to 60 hours per week available for either childcare/leisure or market work at wage w_f. His children are in school for 24 hours per week (9 am to 3 pm) and do not require childcare for that time. If the father would like to work more than 24 hours in the market, he must hire a childcare worker at hourly wage w_c (when drawing budget sets, feel free to assume that $w_c < w_f$).

a. Draw the father's budget set with consumption (*C*) on the vertical axis and (*L*) on the horizontal axis. Precisely label all slopes, intercepts, and points where something changes.
b. Assume standard preferences, and draw the father's labour supply curve. Precisely label all slopes, intercepts, and points where something changes.
c. For the remainder of this question, assume that the childcare worker is unavailable unless paid for at least 10 hours per week. That is, he gets paid

max(h,10)*w_f to work h hours. Draw the father's budget set with consumption (C) on the vertical axis and (L) on the horizontal axis. Precisely label all slopes, intercepts, and points where something changes.

d. Assume standard preferences, and draw the father's labour supply curve. Precisely label all slopes, intercepts, and points where something changes.
e. For the remainder of this question, assume that the father's employer offers a nonwage benefit worth B to all employees that work at least 40 hours per week. Draw the father's budget set with consumption (C) on the vertical axis and (L) on the horizontal axis. Precisely label all slopes, intercepts, and points where something changes.
f. Assume standard preferences, and draw the father's labour supply curve. Precisely label all slopes, intercepts, and points where something changes.

Answer Key

True, false or uncertain (5 points each)

1. True.
2. False. Since leisure is a normal good, Canadians would respond to higher income by increasing their consumption of leisure and thus decreasing their labour supply.
3. False or uncertain. The underlying problem here is that provinces may have other differences in labour market conditions that are statistically related to the generosity of their social assistance programs.
4. True. Some workers may stay employed longer than they would have otherwise in order to be eligible for EI benefits.
5. False. Because of its market power in the product market, we would expect ICBC's labour demand to be *less* elastic than that of most other firms.
6. False. The labour supply of men is quite *inelastic*.
7. Uncertain. It depends on whether these are substitutes or complements for farm labour. The farmer may respond to more costly equipment by moving from capital-intensive to labour-intensive production methods (increasing labour demand and thus wages), or it may respond by significantly reducing production (reducing labour demand and thus wags). Another way of saying this is it depends on whether the substitution effect is larger or smaller than the scale effect.
8. False. Nonwage benefits have grown over the past 50 years both in absolute terms and as a proportion of total compensation.
9. False. It is *non*recurring quasi-fixed costs that lead to labour hoarding.
10. True.

Problems

1. **(26 points)**
   a. (3 points) The elasticity of the firm's output with respect to labour is:

$$\eta_Q = \frac{dQ}{dN} * \frac{N}{Q} = \mathrm{a}zN^{a-1}K^{1-a} * \frac{\mathrm{N}}{\mathrm{aN^aK^{1-a}}} = \mathbf{a}$$

   b. (3 points) The firm's marginal revenue product of labour (*MRPL*) is:

$$MRPL = \boldsymbol{paz}\boldsymbol{N}^{\boldsymbol{a-1}}\boldsymbol{K}^{\boldsymbol{1-a}}$$

   c. (3 points) The firm chooses labour demand so that:

$$w = MRPL = pazN^{a-1}K^{1-a}$$

Solving for N we get:

$$N^D = \left(\frac{\boldsymbol{paz}}{\boldsymbol{w}}\right)^{\left(\frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{a}}\right)} \boldsymbol{K}$$

d. (3 points) The elasticity of labour demand with respect to wages is:

$$\eta_D = -\frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{a}}$$

e. (3 points) The elasticity of labour demand with respect to the output price is:

$$\eta_p = \frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{a}}$$

f. (3 points) The elasticity of labour demand with respect to capital is:

$$\eta_K = \mathbf{1}$$

g. (4 points) The marginal rate of technical substitution (MRTS) is:

$$MRTS = \frac{dQ/dN}{dQ/dK} = \frac{\mathrm{azN^{a-1}K^{1-a}}}{(1-\mathrm{a})\mathrm{zN^{a}K^{-a}}} = \frac{\mathbf{a}}{\mathbf{1}-\mathbf{a}} * \frac{\mathbf{K}}{\mathbf{N}}$$

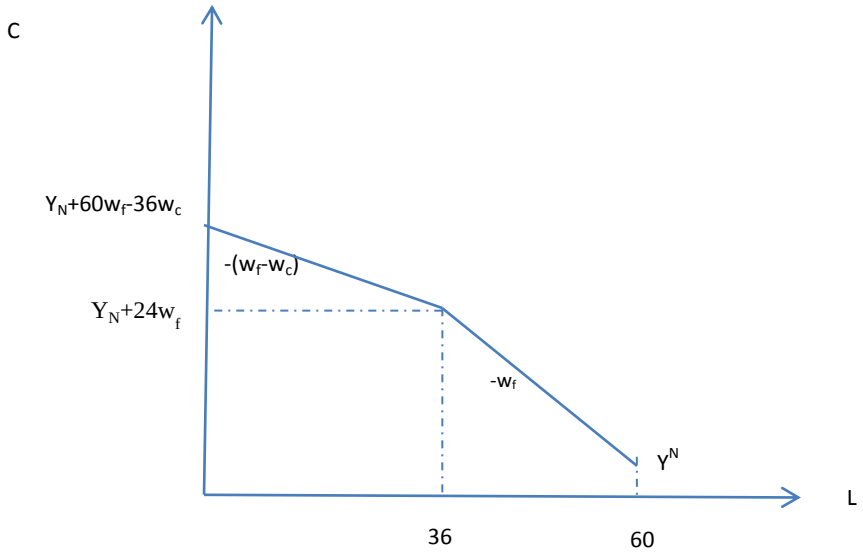
h. (4 points) Cost minimization requires that

$$\frac{w}{r} = MRTS = \frac{\mathrm{a}}{1-\mathrm{a}} * \frac{\mathrm{K}}{\mathrm{N}}$$

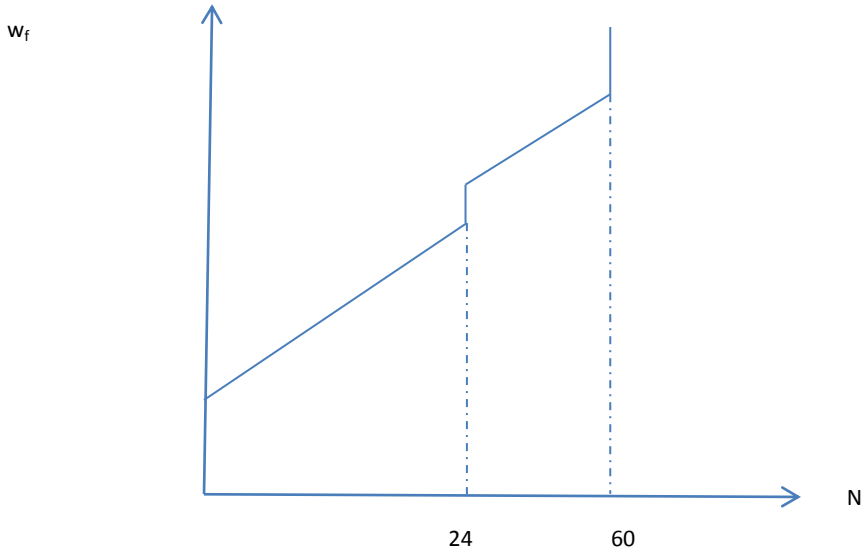
Solving for capital per worker we get:

$$\frac{K}{N} = \left(\frac{\boldsymbol{w}}{\boldsymbol{r}}\right) * \left(\frac{\mathbf{1}-\boldsymbol{a}}{\boldsymbol{a}}\right)$$

2. **(24 points)** Parts b, d, and f of this problem were aimed at seeing if you could take the next logical step in deriving the implications of a particular environment for labour supply decisions. I was not expecting most students to do well on these parts; every exam should have a mix of questions that distinguish the C students from the D/F students and questions that distinguish the A+ students from the A- students.
   a. (7 points) The budget set looks like this:

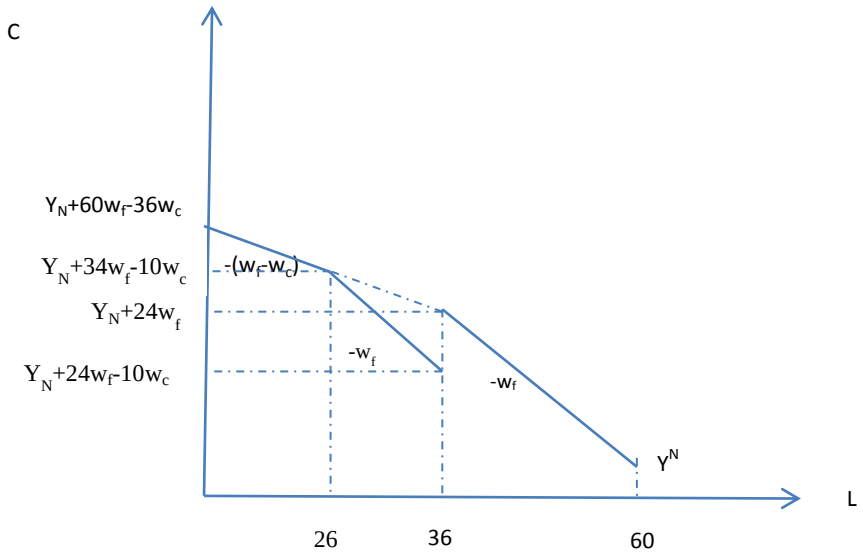


   b. (4 points) The father's labour supply curve looks something like this. Yours may not look exactly like this. The features I am looking for is the presence of a reservation wage (the wage below which labour supply is zero), and some sort of a sharp reduction in elasticity at 24 hours (since the opportunity cost of leisure goes from w_f to w_f-w_c) and near 60 hours (since his labour supply cannot go above 60).

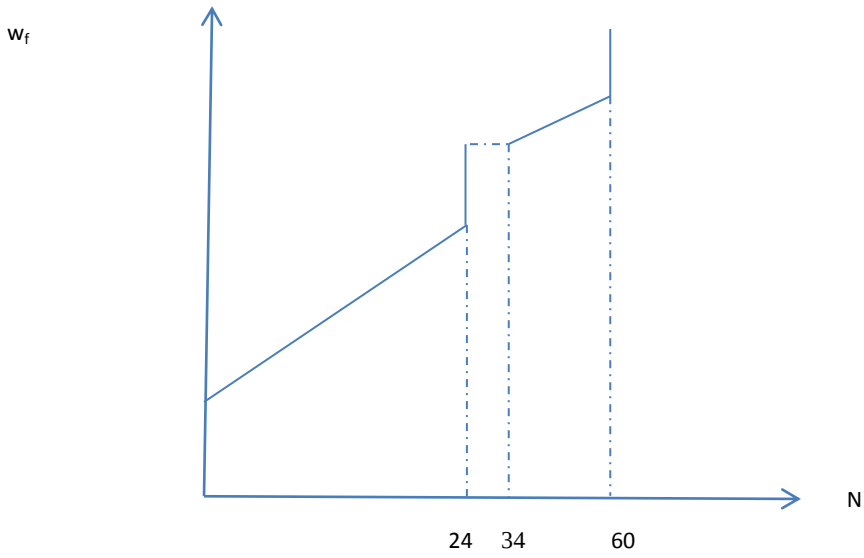


One common and unfortunate mistake was to assume that the reservation wage would be equal to non-labour income Y_N. There is no reason for this to be the case, especially since non-labour income is measured in different units (dollars) than the wage (dollars per hour worked). If you receive $1,000 in nonlabour income does this mean you will have a reservation wage of $1,000 per hour?

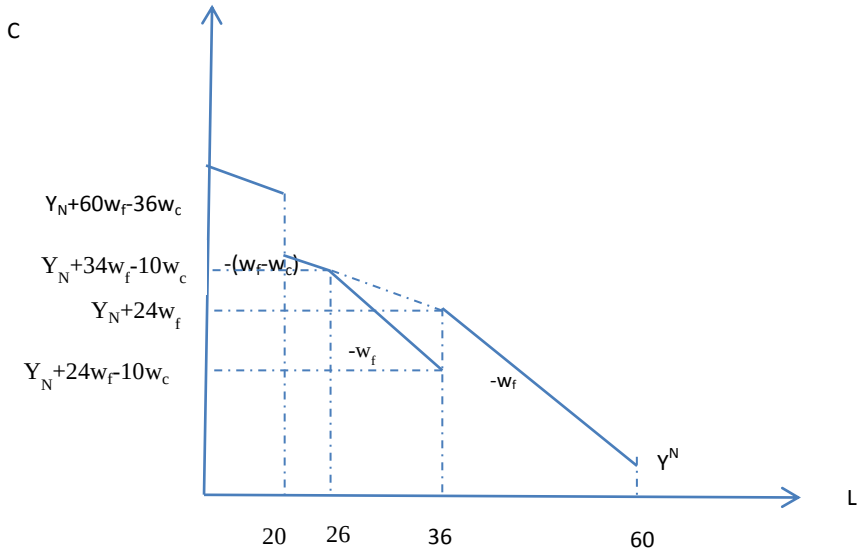
c. (5 points) The budget set looks like this:



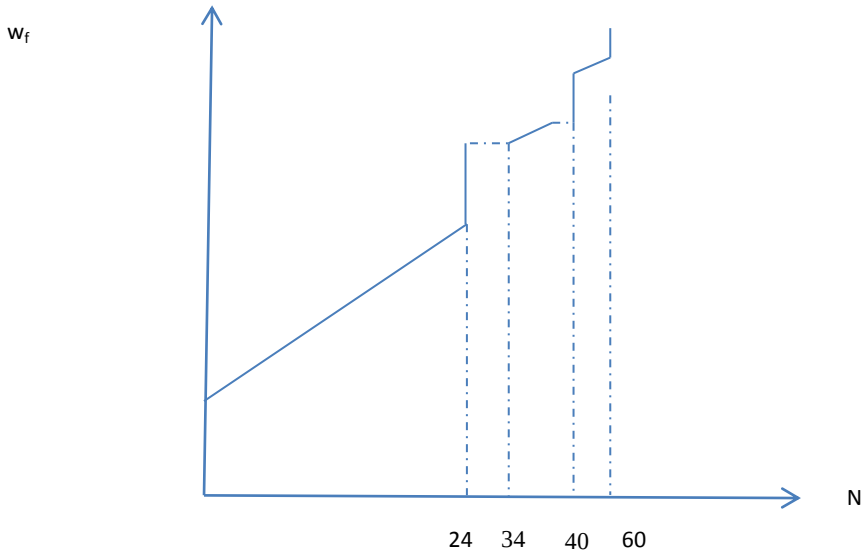
d. (3 points) The father's labour supply curve looks something like this. Yours may not look exactly like this. The new feature I am looking for is that labour supply will stay stuck at 24 hours (because any small increase in labour supply means he has to pay for 10 hours of childcare), and then will at some point jump up to 34 hours (once it is worth it to pay for at least 10 hours of childcare).



e. (4 points) The budget set looks like this:



f. (3 points) The father's labour supply curve looks something like this. Yours may not look exactly like this. The new feature I am looking for is that labour supply will suddenly jump up to 40 hours (to obtain the benefit) at some wage. Depending on the benefit, it's not clear at what wage that will occur, and so this new feature may end up eliminating the other features.


w_f
N
24
34
40
60